

Deformable Subreflector Computed by Geometric Optics

M. S. Katow and I. Khan

Ground Antenna and Facilities Engineering Section

W. F. Williams

Radio Frequency and Microwave Subsystems Section

Using a Cassegrainian geometry, the 64-meter antenna with its distorted paraboloidal reflecting surface can be forced to produce a uniform phase wavefront by a pathlength-compensating subreflector. First, the computed distortion vectors at the joints or nodes of the main reflector structure supporting the surface panels are best fitted to a paraboloid. Second, the resulting residual distortion errors are used to determine a compensating subreflector surface by ray tracing using geometric optics principles. Third, the totally corrected subreflector surface is defined by the normal directions and distances to the surface of the original symmetric hyperboloid for the purpose of evaluation. Finally, contour maps of distortions of the paraboloid reflector and the compensating subreflector are presented. A field-measured check of the subreflector in focused position as computed by the described methodology is also presented for the antenna position at horizon look with the geometry at 45 degrees elevation.

I. Introduction

In the Cassegrainian radiofrequency (RF) antenna, the spherical wavefront emanating from the RF feed reverses direction at the hyperboloid focus, then it is reflected by the paraboloid reflector into a flat, uniform-phase wavefront to space. Since the diameter of the paraboloid is generally about ten times larger than the hyperboloid (and therefore more massive), its surface errors can introduce much larger distortions to the reflecting RF rays and hence introduce larger pathlength differences to the desired planar wavefront. The subreflector surface errors are usually minor or negligible in comparison.

Using a geometric-optics based ray-tracing technique in a subreflector-forming (SUBFORMING) computer program, the RF pathlength error introduced by the paraboloid's distortion may theoretically be compensated for at the subreflector's surface for each defined ray.

The 64-meter antenna reflector structure (at DSS 14), with structural brace modification included, has been modeled using NASTRAN computer analysis to compute distortions caused by environmental loads, e.g., gravity loading due to elevation angle motion. The distortions of the structural joints of the forty-eight main radial ribs (Ref. 1) were computed for the gravity loading at the antenna's horizon position

(elevation angle = 0) with the surface panels previously set to the design paraboloid at a 45-deg elevation angle. A paraboloid was best fitted to these distortions using the RMS program (Ref. 2) and the resulting antenna variables were used in the SUBFORMING program and evaluated by the NORM program.

II. Analysis Steps

The input data required by the geometric optics-based program SUBFORMING (see Appendix A) are delineated in Fig. 1. The main reflector structure's computed distortions in Cartesian coordinates ΔX , ΔY , and ΔZ are added vectorially to the undeformed coordinates of the reflector structure to define the distorted shape. A new best-fit paraboloid is then determined for this distorted shape using the RMS program (Ref. 2), which defines a new coordinate system for the new paraboloid (which has been rotated and translated relative to the original). The rotation-translation of the best-fit paraboloid is indicated in Fig. 1 by $ZETA$ (rotation) and HZ, HY (translations).

The RF feed phase center is defined in Fig. 1 by PDSC. The RF feed phase center and the focus of the best-fit paraboloid have deflected from gravity loading, assuming that the hyperboloid system is focused axially and laterally. The hyperboloid focal length FCO has changed to FC due to the deflections of the RF feed and the change in focal length of the paraboloid.

The original nodes of the paraboloid (before the distortions ΔX , ΔY , and ΔZ) were located on radial lines equally spaced at 7.5 deg (ψ) around the central axis and in approximately equal intervals (9 points) along the radials. However, with distortions and surface-slope changes resulting from gravity loading, a ray parallel to the axis of the best-fit paraboloid will not necessarily stay in the plane defined by the ray and the hyperboloid's axis as the ray reflects from the distorted surface of the paraboloid. When this reflected ray impinges on a symmetric hyperboloidal subreflector and is again reflected, it will miss the RF feed phase center. Therefore, a compensating reflecting surface on the subreflector will be required to maintain focus.

The SUBFORMING program computes points P on the compensating subreflector, thus maintaining the equal path-length requirements for all rays, as described in Appendix A. In a spherical coordinate system as delineated in Fig. 2, the program outputs the radius R and the angles θ and ϕ to locate points P and the vector normal to the surface of the compensating subreflector at points P .

One assumption was made at this time: the normal to the compensating subreflector lies in the ϕ -plane, although SUBFORMING may not generate this answer for distorted paraboloids. This reduces the solution of the normal corrections with respect to the design hyperboloid parameters to a two-dimensional problem. The normal corrections computed for each ϕ value could then be used to plot the surface contours with minor errors.

Referring to Fig. 3, the equation of the normal correction, PS , can be developed from the data described in Fig. 2 (see Appendix B). The normal's intercept point, S , on the hyperboloid is computed and the normal length PS (required correction) is then computed.

The above described steps were incorporated in the computer program NORM with an added provision to translate points P parallel to the symmetric axis OQ of Fig. 3 to simulate the focusing motion along the symmetric axis of the hyperboloid.

The computed normal corrections may be contour plotted using JPL's plotting subroutines where the intersection points of the normals and the hyperboloid are not required to be uniformly spaced.

III. Solution Verification

One check of the methodology was made by considering the 64-meter antenna reflector data with no distortions as inputs to the SUBFORMING program using the design values (Fig. 3) for the 45-deg elevation as follows:

Paraboloid focal length:	26.0933 m (1027.294 in.)
Hyperboloid parameters:	$a = 4.5219$ m (178.029 in.)
	$b = 5.1560$ m (202.992 in.)
	$c = 6.8580$ m (270.000 in.)

The answers from the SUBFORMING program were input to the NORM program; the largest normal corrections were found to be 0.0015 mm (0.00006 in.). In other words, the SUBFORMING program's output of points P was verified to be accurate enough for the design hyperboloid.

A. Check On Antenna Horizon Position

In antenna operation, the reflective surfaces are "rigged" or set at 45 deg elevation angle. As the antenna rotates to the horizon position, the main paraboloidal reflector not only distorts, but also changes in focal length as described by the best-fit paraboloid. The prime focus translation requires focusing motion of the design hyperboloidal subreflector as well as

shape changes to maximize performance. By use of the SUBFORMING program, the original design hyperboloid can be proven to be in perfect focusing position for the shorter focal length best-fit paraboloid of the horizon position, if the distance a of Fig. 3 is correct.

The computed parameters after the best-fit of a paraboloid to the distortions at horizon position are shown in Fig. 4 with the contour map of the normal errors pictured in Fig. 5 (generated by the RMS program).

First, it was necessary to locate the position a of the design hyperboloid at focused position. This was accomplished by reducing the distortion of the best-fit paraboloid to zero. Second, the curve of the subreflector generated by the SUBFORMING program was matched exactly to the original design hyperboloid by translating the generated subreflector until the normal corrections were reduced to very small values. This was accomplished by iteration of X_1 of Fig. 3 in the NORM program, and by changing the value of a .

Finally, by using distances a and x_1 (computed above), the horizon position distortions were added to the SUBFORMING program and the deformed shapes of the compensating subreflector (as described by normal corrections to the design hyperboloid) were then contour plotted in Fig. 6. As expected, the normal corrections on the hyperboloid are the exact picture of the paraboloid's distortion as given by Fig. 5.

B. Field Verification

A comparison of the focused position of the 64-meter antenna hyperboloid computed in this article and field data is made. Figure 7 illustrates the computed hyperboloid focus travel from the 45-deg setting position to the horizon position, which is 2.11 cm (0.83 in.).

Figure 8 (Ref. 3) presents the field data where a rather large extrapolation is required to determine that the subreflector indicated movement is (2.54 cm -0.58 cm) or 1.96 cm (0.77 in.). Since the field-indicated position of the subreflector is obtained from a revolution counter on the jackscrew drive of the subreflector's supporting system, the deflections of the

jackscrew mechanism and the supports of the subreflector are lost. If some of the above noted lost deflection is added to the measured travel of 1.96 cm (0.77 in.), the field data will compare closely to the computed 2.11-cm (0.83-in.) travel.

It should be noted that there has been a lack of consistency of the indicated subreflector position between different RF feeds; this inconsistency has not been resolved to date. A possible reason involves the RF beamshaping peripheral flange, fitted around the hyperboloidal portion of the subreflector. This flange is effective in reducing feed spillover power beyond the paraboloid edge, and is optimum for S-band. The 30.48-cm (12-in.) radial distance of the flange (about 2λ at S-band) is not optimum at other (particularly higher) frequencies. We believe that these field-measured subreflector positions for the antenna maximum gain are influenced by phase best fitting, of the feed wavefront ripples near the paraboloid rim.

IV. Summary

- (1) The duplication of the distortions of the main paraboloidal reflector and the compensating subreflector by contour maps indicated accurate solutions by the SUBFORMING program. For improving the performance of a Cassegrain antenna, the subreflector surface can be altered mechanically as the antenna rotates from its "rigged" or setting elevation position.
- (2) The mathematical methods used in the SUBFORMING program were checked for accuracy by: (a) generating a perfect hyperboloidal subreflector for a perfect paraboloidal main reflector, and (b) determining the new focus of the original design hyperboloid for a paraboloid with a changed focal length that checks the field measured value for the 64-meter antenna.
- (3) Deforming the subreflector of the 64-meter antenna to match the bumps over the elevation bearing will be difficult to implement because: (1) the sharp curvature changes will require stretching, and (2) additional stiffness is required of the subreflector to satisfy operational specifications in a high-wind environment.

References

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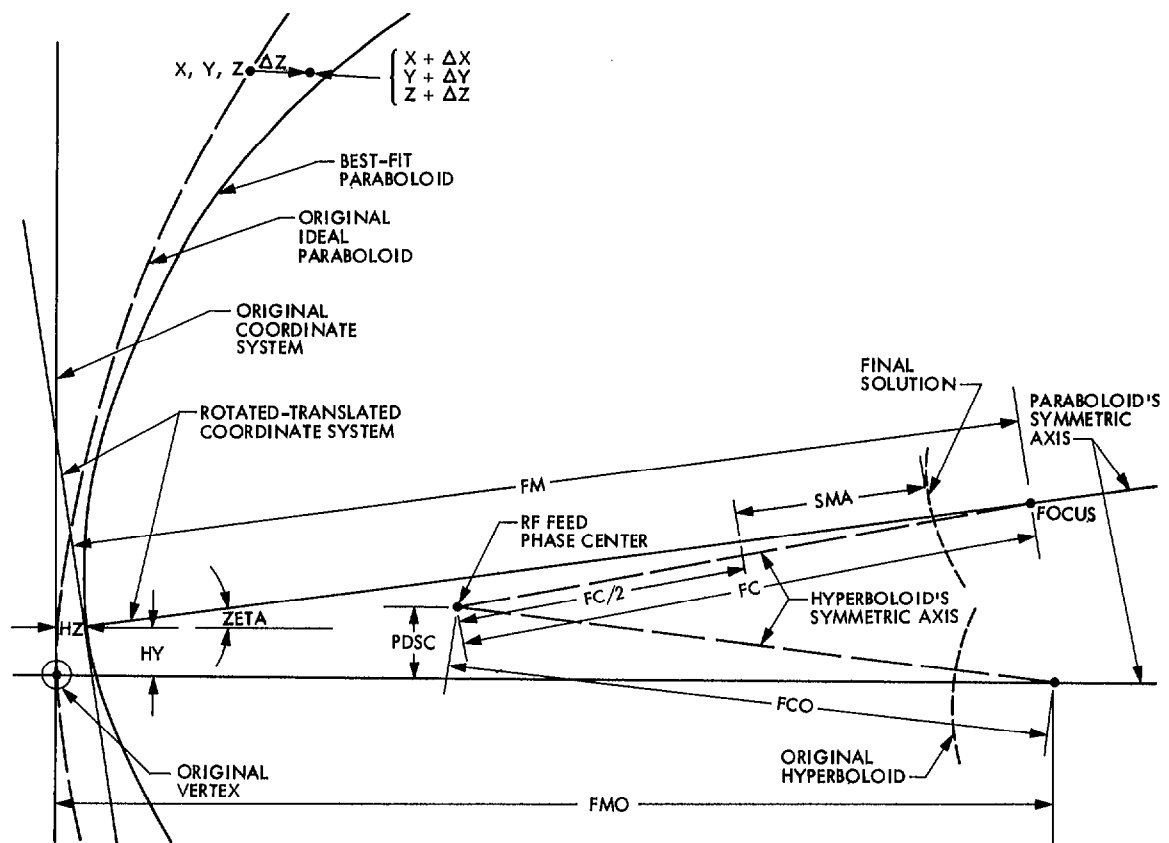


Fig. 1. Best-fit paraboloid's coordinate system

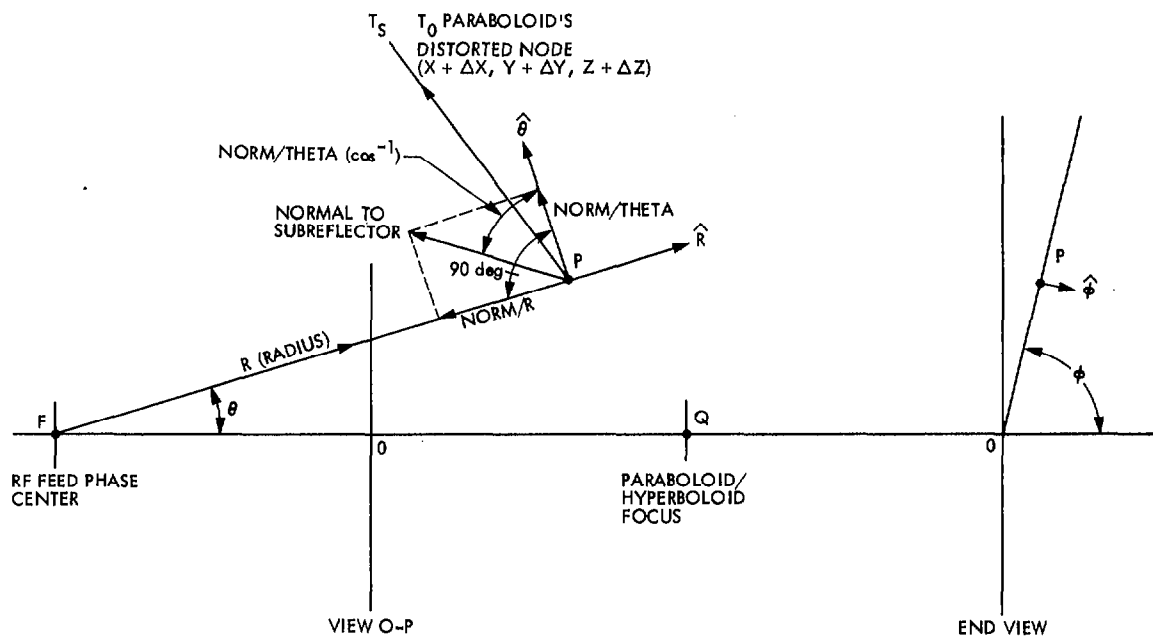


Fig. 2. RF rays coordinate system at feed phase center

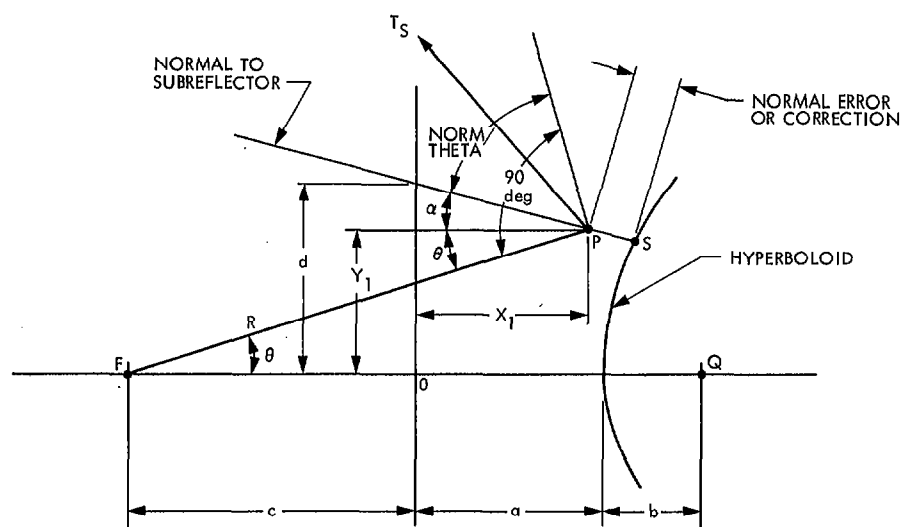


Fig. 3. Normal error or correction in ϕ -plane

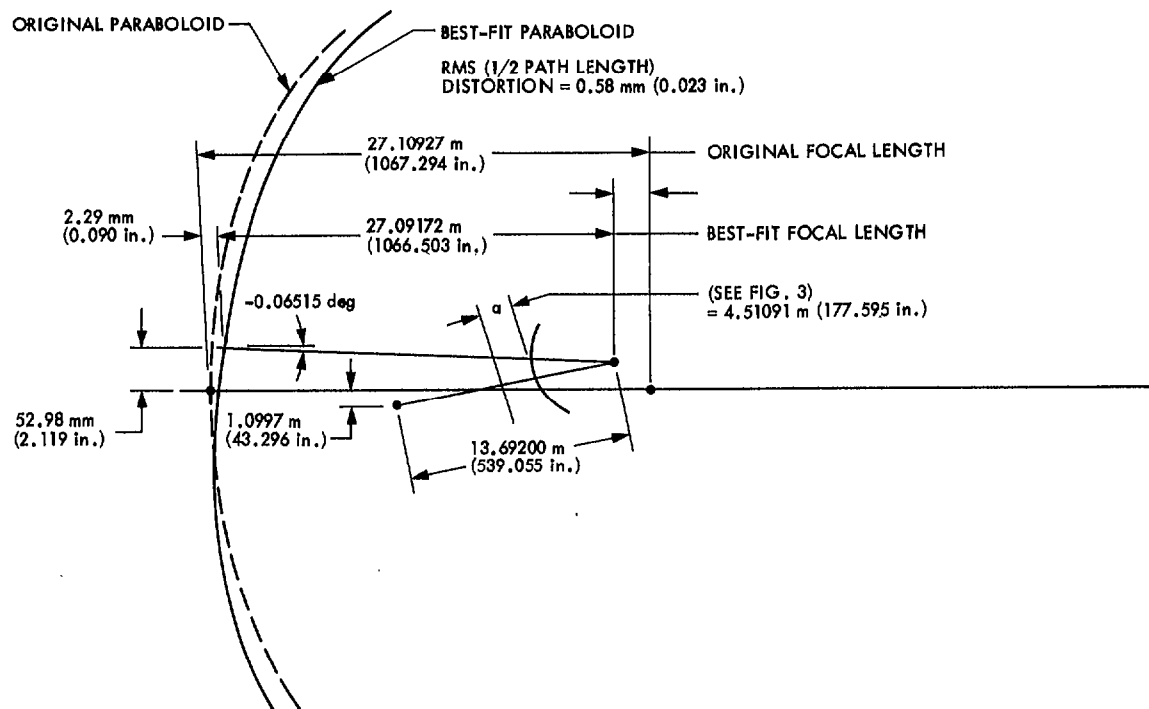


Fig. 4. Horizon position best-fit parameters (surface panels set at 45 deg)

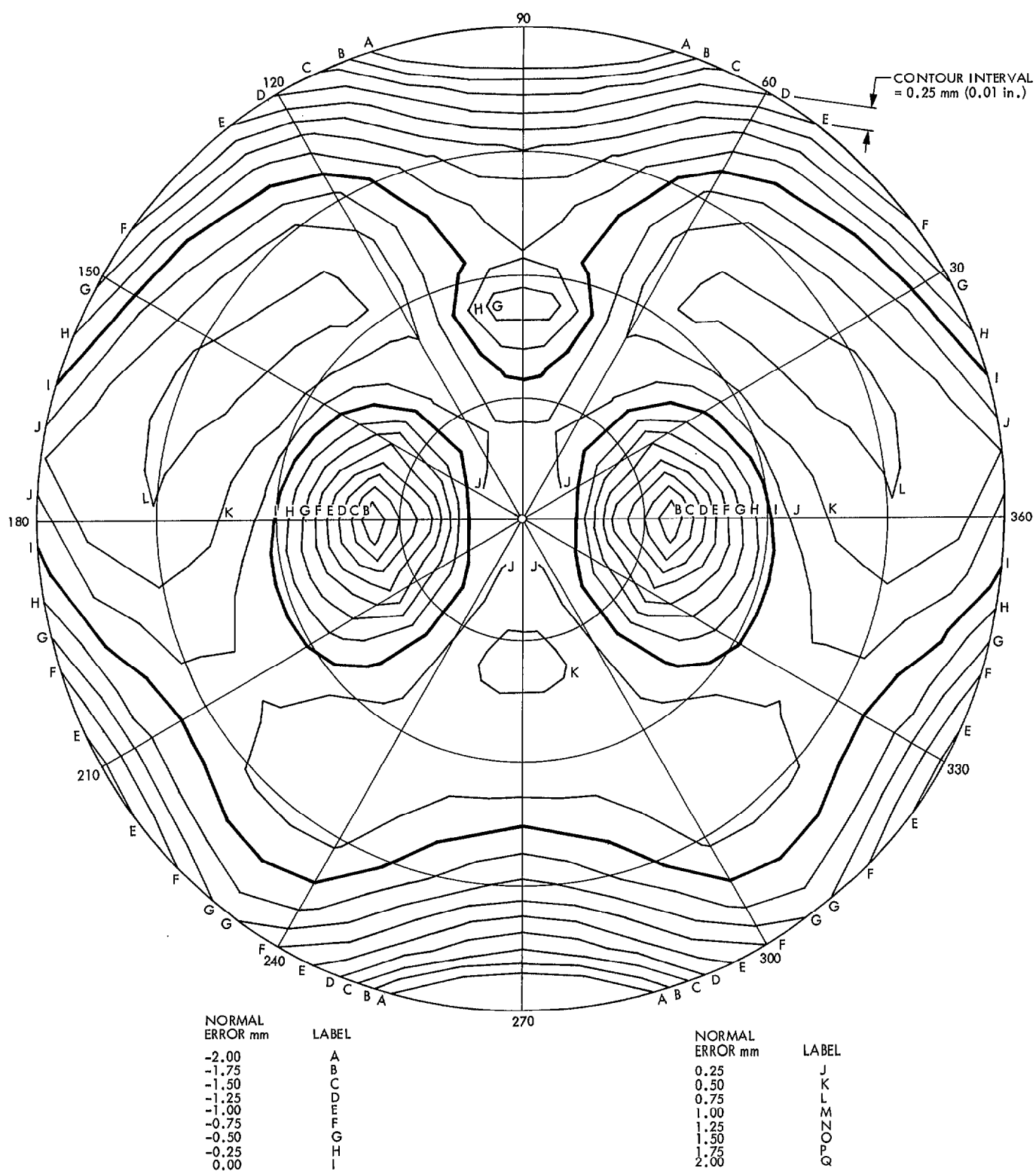


Fig. 5. 64-meter paraboloid distortion contour map-after best-fit at horizon position (surface panels set at 45 deg)

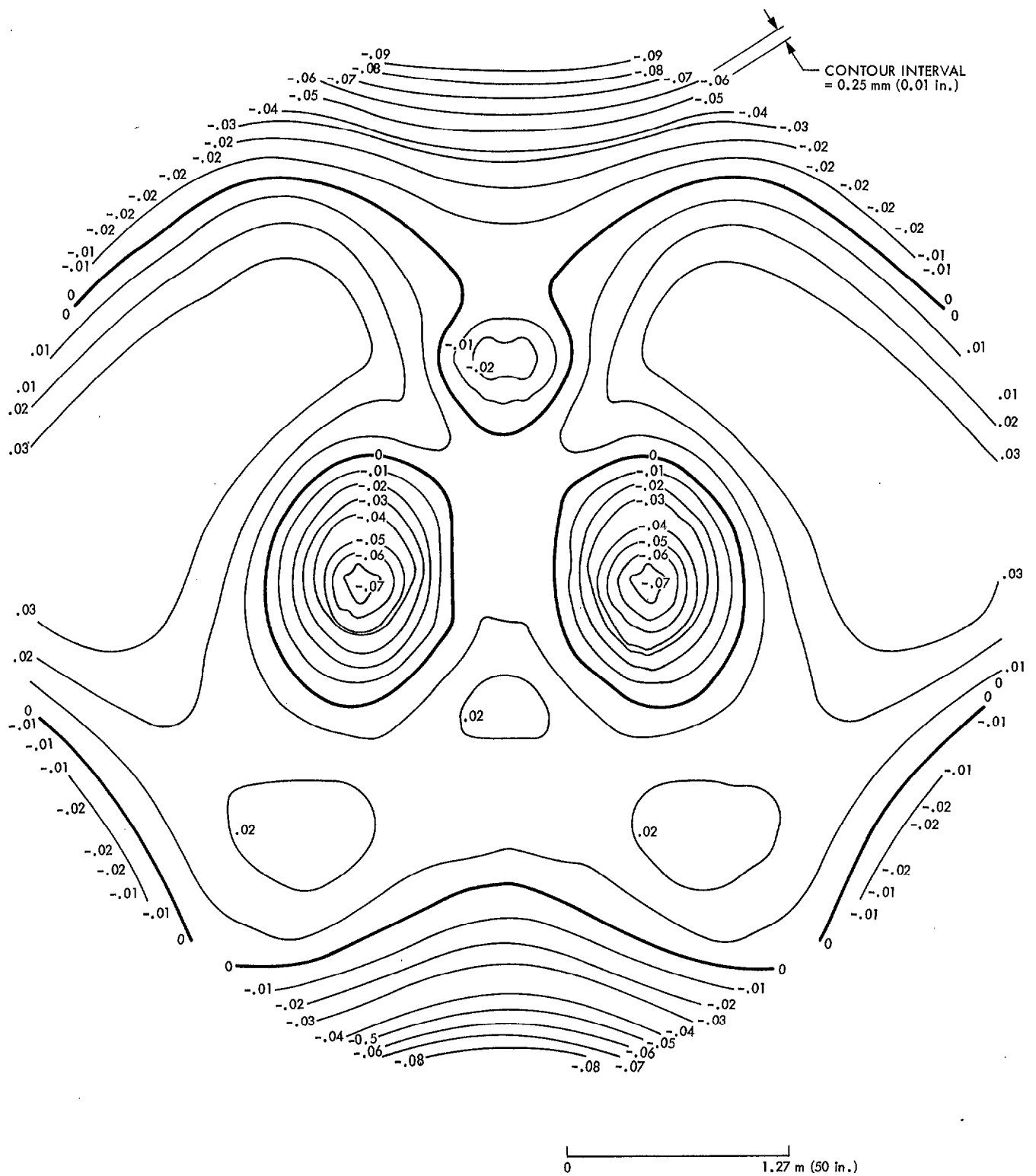


Fig. 6. Deformed compensating subreflector contour map (horizon position, setting at 45 deg)

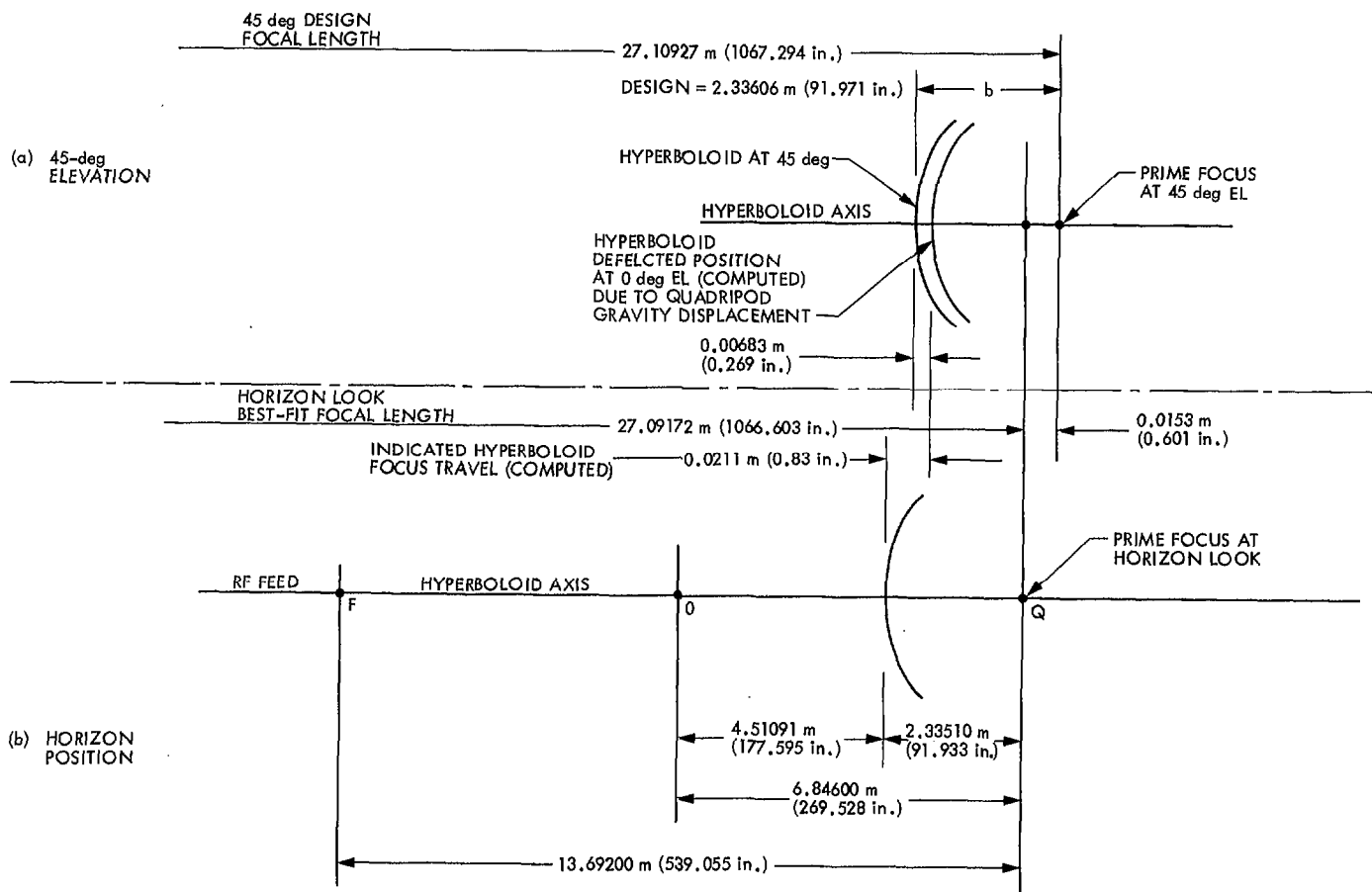


Fig. 7. Computed focus travel of the hyperboloid from 45-deg elevation to horizon positions

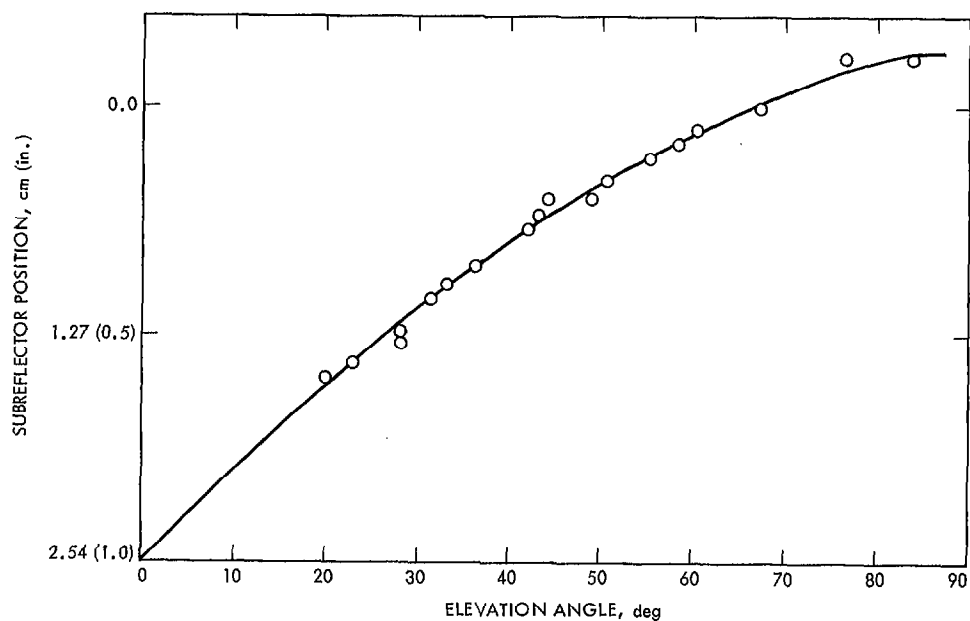


Fig. 8. Indicated subreflector position vs. elevation angle (field readings)

Appendix A

The Subforming Program

I. Theory of Operation

A computer program has been developed, using geometric optics, for synthesizing the needed deformation of a subreflector to compensate for a deformed main reflector, maintaining RF path lengths and hence eliminating phase loss. This synthesis depends upon three factors to be determined about each main reflector point. These are: (a) the point, X , Y , and Z , (b) partial derivatives at each point, $\partial/\partial Z$, $\partial/\partial \rho$ and $\partial/\partial \psi$ (choosing a cylindrical system) and, (c) a chosen pathlength that must be held constant. The technique for doing this is as follows: at a main reflector point, (deformed), determine the slopes of the surface, $\partial/\partial \rho$, $\partial/\partial Z$ and $\partial/\partial \psi$ (in cylindrical coordinates), and from this determine the components of the normals, n_x , n_ρ , n_ψ . The direction of the incoming rays is known, assumed in the $-Z$ direction, and hence the reflected ray direction can be determined. The direction cosines for the reflected ray are then calculated. This reflected ray must intersect the unknown (to be calculated) deformed subreflector and be reflected to the RF feed point, or focus. Total distances for all rays must be equal and constant. This pathlength is somewhat arbitrary. The length used in the SUBFORMING program is the distance from the paraboloid aperture along the axes to the paraboloid vertex, back to the original hyperboloid vertex and back to the feed focus (or phase center). The ray direction and this pathlength requirement represent sufficient information to determine the subreflector points, (X_S, Y_S, Z_S) . The reflected ray will also determine the components of the normal and the slopes, $\partial Z_S/\partial X_S$ and $\partial Z_S/\partial Y_S$. This same data is used to calculate the points in the spherical coordinate system as is generally desired. (Ref. 4)

Performing these operations upon a paraboloid that is deformed in some closed-form analog manner is fairly straightforward, e.g.,

$$\Delta Z = A \rho \cos N \psi$$

Partial derivatives can be evaluated at as many points as desired; normals and direction cosines and hence ray directions are determined and a matching subreflector point is obtained corresponding to the main reflector data point. However, with a real system (the 64-meter, DSS 14 antenna), deformation data is presented in a regular fashion as digital error points along approximately equal radial distances from the antenna vertex and at approximately equal angles around the azimuth direction ψ . No data regarding slopes are available and so a method should be found to determine these required func-

tions. The 432 distinct points are available (from DSS 14 data) and the appropriate slopes are found for each point. The approach chosen is to place a best-fit polynomial surface through nine points, eight of them surrounding the central point, as indicated in Fig. A-1. In this two-dimensional figure, the dimension Z is not indicated, only the cylindrical coordinates ρ and ψ . The best-fit surface is theoretically determined using available JPL computer subroutines. A separate surface is obtained for each deformed point of the paraboloid since the local surface determination will be at its best for the one central point. Exceptions are for the last points (farthest) out on the antenna periphery (rim) and the innermost points near the vertex. In these cases, slopes are determined using the surface that is generated about the next adjacent point on the radial.

The best-fit surface generated for each deformed point will be represented by a quadratic equation $Z = g(\rho, \psi)$ with six constants determined by the subroutine. Partial derivatives are then determined and evaluated. Normals and direction cosines are calculated. The subreflector point (X_S, Y_S, Z_S) is determined as a point on the ray path which, with reflection, will fulfill the chosen equal pathlength requirement. The direction of the ray reflection is used to calculate subreflector slopes and normals. Spherical and/or cylindrical coordinates are obtainable directly from the Cartesian solution.

Use of the computer program is described in the following paragraphs. Input parameters are as shown in Fig. 1.

II. Program Structure

Distortion data are input as error positions, ΔX , ΔY , and ΔZ , from the main reflector coordinates, X , Y , and Z . These distortion points must be presented in a sequence that: (1) proceeds from an inner hub position, (2) proceeds out along an approximate radial line to a final position on this radial, (3) proceeds to a next-adjacent radial starting at the hub, and again proceeds out this radial, and (4) continues around the reflector surface until completed. Angular positions between radials should be approximately equal as should the radial spacing between listed radial points, for improved program accuracy. The program is set to accept up to 15 points on any radial position and 60 radial lines, i.e., 6 deg apart, for a total of 900 distortion points. These distortion points will generally be defined relative to an "ideal" paraboloidal reflector with a coordinate system, which has its origin at

the vertex and its Z -axis through a focal point. A best-fit paraboloid is recognized as a better fit to the distorted system. In this case, it is desirable to define a new coordinate system according to this best-fit paraboloid vertex and focal length. This coordinate system is input to the program.

The solution is a set of output points P defining a subreflector contour. Each point matches, on a one-to-one basis, a point on the distorted paraboloid. The slope of and the normal to the surface are determined for each point. The data appear on three files when the program is executed. The first file contains the input data concerning the main reflector, the second file contains subreflector data in spherical coordinates, and the third file contains the same data in Cartesian coordinates. These files can then be recorded in a data file, punched or placed on temporary files, and printed.

III. Inputs

1. Title = your choice
2. FMO = focal length of original "ideal" paraboloid
3. FCO = focal length of original hyperboloid
4. FM = focal length of chosen "best-fit" paraboloid, can be the original focal length, FMO .
5. D = diameter of main reflector
6. FC = focal length of chosen subreflector, i.e., distance from feed focus to the focal point at FM , could be FCO

7. SMA = the dimension "a" of a hyperboloid, taken from the chosen vertex of the subreflector to the origin of $FC/2$
8. $PDSC$ = feed displacement relative to the "ideal" paraboloid reflector axis in Y direction only
9. HZ = best-fit vertex translation in Z direction, taken from original coordinate system origin (may be 0.0)
10. HY = as HZ , but in Y direction
11. $ZETA$ = best-fit axis rotation, relative to original axis, in $Y-Z$ plane only (may be 0.0)
12. $JMAX$ = Number of data points on one radial; must be equal for all radials
13. $KMAX$ = Number of radial lines
14. $\left. \begin{matrix} X(J, K) \\ Y(J, K) \\ Z(J, K) \end{matrix} \right\}$ = Coordinates of point (J, K) on the original "ideal" reflector.
15. $\left. \begin{matrix} DX(J, K) \\ DY(J, K) \\ DZ(J, K) \end{matrix} \right\}$ = Distortion magnitude in the X , Y , and Z directions to the final reflector

IV. Output

The printed output consists of the listed input main reflector (deformed) data in the new coordinate system and the solution for the subreflector in spherical coordinates in this same coordinate system.

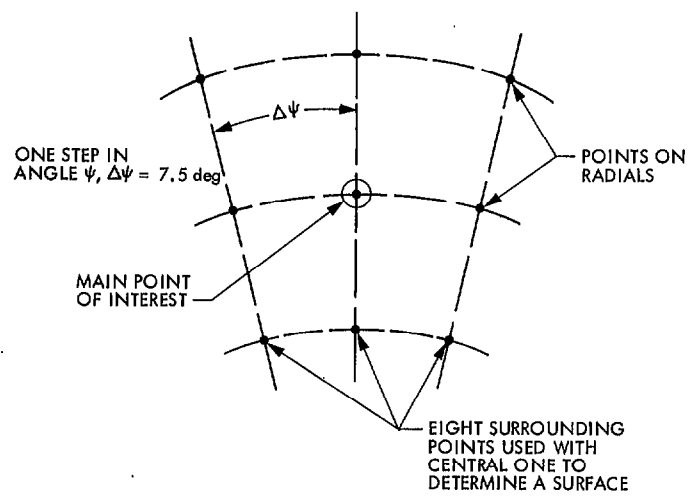


Fig. A-1. Nine points best-fit polynomial surface

Appendix B

NORM Program

Referring to Fig. 3, the equation for the normal PS was developed, and after its intercept to the hyperboloid (two dimensional analysis in plane ϕ , see Fig. 2) was calculated, its length between the point P and S was computed. The equation of the normal is

$$Y = mX + d \quad (B-1)$$

where

$$X_1 = R \cos \theta - 270$$

$$Y_1 = R \sin \theta$$

$$a = -90 + \cos^{-1} (\text{norm}/\theta) + \theta$$

$$m = \tan a$$

$$d = Y_1 - X_1 \tan a$$

Substituting above values in Eq. (B-1),

$$X = (TA)Y - C1 \quad (B-2)$$

where

$$TA = 1/\tan a$$

or

$$\frac{Y_1}{\tan a} - X_1 = C_1$$

The hyperbola equation is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (B-3)$$

Substituting equation (B-2) into (B-3) for X and solving the resulting equation by the binomial equation, the intercepts on hyperbola become

$$Y_H = \frac{-CB \pm \sqrt{CB^2 - 4 \cdot CA \cdot CC}}{2 \cdot CA}$$

$$X_H = \sqrt{\frac{a^2 b^2 + a^2 Y_H^2}{b^2}}$$

where

$$CA = TA^2 - \frac{a^2}{b^2}$$

$$CB = -2TAC1$$

$$CC = C1^2 - a^2$$